

Neutrino propagation in a medium with a magnetic field

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Abstract

We study the properties of neutrinos propagating in an isotropic magnetized medium in the two physical approximations of degenerate Fermi gas and classical plasma. The dispersion relation shows that, for peculiar configurations of the magnetic field, neutrinos can propagate freely as in vacuum, also for very large density; this result can be very important in the study of supernova evolution. For mixed neutrinos, the presence of a magnetic field can alter significantly MSW oscillations, and for particular configurations of the field the resonance condition no longer occurs. Furthermore, on the contrary to that happens in non-magnetized media, spatial dispersion arises and neutrino trajectory can be in principle deviated; however a simple estimate shows that this deviation is not detectable.

1 Introduction.

When a neutrino passes through a medium, the interaction with the particles in the background gives rise to modification of the properties of the neutrino itself. For example, even if neutrinos are exactly massless, in a medium they can acquire an effective mass [1] and also an effective electromagnetic coupling [2]. The most famous effect for massive and non degenerate neutrinos is the MSW effect [3]: if the resonance condition is fulfilled, for neutrinos (created by weak interactions with a definite flavour) propagating in matter, the probability for transition to another flavour can be appreciable different from zero also for very small mixing angles. The solution of the solar neutrino problem [4] in terms of this effect [5] is a well known tool.

The covariant formalism that enables one to consider the interactions of a neutrino passing through a medium is the Finite Temperature and Density Quantum Field Theory [6] (throughout this paper we use the real-time formulation [7] of this theory, in which the Feynman rules for the vertices are identical to the corresponding ones in the vacuum). The effect of the temperature and of the density is taken into account in the expressions of the free particle propagators. For fermions and bosons we have respectively [7]:

$$S_F(P) = (\not{P} + m) \left[\frac{1}{P^2 - m^2} + i \Gamma_F(P) \right] \quad (1)$$

$$D_{\mu\nu}(P) = -g_{\mu\nu} \left[\frac{1}{P^2 - m^2} - i \Gamma_B(P) \right] \quad (2)$$

where

$$\Gamma(P) = 2\pi \delta(P^2 - m^2) [\theta(P \cdot u) n(P) + \theta(-P \cdot u) \bar{n}(P)] \quad (3)$$

and

$$n_F(P) = \frac{1}{e^{\beta(|P \cdot u| - \mu)} + 1} \quad n_B(P) = \frac{1}{e^{\beta(|P \cdot u| - \mu)} - 1} \quad (4)$$

are the Fermi-Dirac and Bose-Einstein distribution functions ($n_{\bar{F}}$ and $n_{\bar{B}}$ are the distribution functions for the antiparticles). Note that in a medium another 4-vector must be considered: the medium 4-velocity u_μ . We restrict ourselves to temperature which are low compared with the W^\pm and Z^0 masses, so that we don't consider the temperature-dependent term $\Gamma_B(P)$ in $D_{\mu\nu}(P)$.

In the present work we want to study the propagation of a neutrino in a magnetized medium. An external magnetic field forces the background particles to describe spiral-like trajectories along the force lines, so that it influences deeply the behaviour of the medium. Consequently, the interaction of neutrinos with the modified background particles modifies neutrino properties both in comparison with the vacuum ones and in comparison with their properties in a non-magnetized medium.

The medium we assume it consists of an electron gas, and a positive charge density is spread homogeneously throughout in order to maintain the charge neutrality (jellium model). Although our following analysis works for a degenerate gas as well as for a non-degenerate gas (whether or not it is relativistic), when the calculations are carried explicitly we discuss the specific cases of an electron gas in condensed matter and in

plasmas. For the former, we shall make the simplification of a gas of free charged “effective” fermions (the electrons dressed by the interaction with ions), uninfluenced by the mutual electrostatic forces. In the latter case we stress the relevance of the long-range electron-electron interaction, that gives rise to a large repertoire of collective behaviour. By crossing from the temperature range of a degenerate gas ($T \ll T_F$ where T_F is the Fermi temperature) to much higher temperatures ($T \gg T_F$), the electron gas can be described in terms of non-quantum mechanics. The interest in considering a plasma as medium allows us to debate possible implications of our results in different physical contexts, i.e. astrophysical field.

In the following section we evaluate the neutrino self-energy in a magnetized medium in the two physical approximation of a degenerate gas and of a classical plasma. The results here obtained are used to study the dispersion relation in section 3 and spatial dispersion in section 4. The modifications on neutrino matter oscillations are, instead, analyzed in section 5. Finally, in section 6 some possible applications of the obtained results are picked out.

2 Neutrino self-energy in a medium with an external static magnetic field.

The propagation of a neutrino, moving along the z-axis with 4-momentum $K_\mu = (\omega, 0, 0, k)$, is described by the Dirac equation, that in momentum space is

$$(\not{K} - m - \Sigma) \psi = 0 \quad (5)$$

where $\Sigma(K)$ is the neutrino self-energy.¹ For propagation in a non magnetized medium [8], the one-loop relevant contribution of the electron background to the self-energy is given by the diagrams in fig.1,2:

$$\begin{aligned} -i\Sigma_W &= \int \frac{d^4P}{(2\pi)^4} \left(\frac{-ig}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5) \right) iS_F(P) \left(\frac{-ig}{2\sqrt{2}} \gamma_\nu (1 - \gamma_5) \right) iD_W^{\mu\nu}(K - P) \quad (6) \\ -i\Sigma_Z &= \left(\frac{-ig}{4\cos\theta_W} \right) \gamma_\mu (1 - \gamma_5) iD_Z^{\mu\nu} \int \frac{d^4P}{(2\pi)^4} (-1) \cdot \\ &\quad \cdot \text{Tr} \left[\left(\frac{ig}{4\cos\theta_W} \right) \gamma_\nu (1 - 4\sin^2\theta_W - \gamma_5) iS_F(P) \right] \quad (7) \end{aligned}$$

where Σ_W contributes only to ν_e self-energy, while Σ_Z contributes also to that of ν_μ, ν_τ . The fermion propagator $S_F(P)$ is taken by eq. (1).

In general, the self-energy can be write as the sum of a temperature-independent part and a temperature-dependent term; we are interested on the effects that the medium can have on neutrino propagation, and then we consider only the temperature-dependent

¹For the sake of simplicity, here we consider a neutrino with a Dirac mass and zero mixing angle; the extension to non-zero mixing angles will be discussed in section 5.

term $\Sigma(T)$. Furthermore, $\Sigma(T)$ will be complex, but we ignore absorptive effects, and consider only the real part of the self-energy. Then, we can write

$$\mathcal{R}[\pm] = \frac{\infty + \gamma_{\nabla}}{\epsilon} \mathcal{R}[\pm] \frac{\infty - \gamma_{\nabla}}{\epsilon} \quad (8)$$

with

$$\mathcal{R}[\pm]_{\mathcal{W}} = -\}^{\epsilon} \int \frac{[\Delta \mathcal{P}]}{(\epsilon \pi)^{\Delta}} \frac{-\mathcal{F}(\mathcal{P})}{(\mathcal{K} - \mathcal{P})^{\epsilon} - \mathcal{M}_{\mathcal{W}}^{\epsilon}} \mathcal{P} \quad (9)$$

$$\mathcal{R}[\pm]_{\mathcal{Z}} = -\frac{\}^{\epsilon}}{\epsilon \mathcal{M}_{\mathcal{Z}}^{\epsilon}} (\infty - \Delta \mathcal{J} \setminus \epsilon \theta_{\mathcal{W}}) \int \frac{[\Delta \mathcal{P}]}{(\epsilon \pi)^{\Delta}} -\mathcal{F}(\mathcal{P}) \mathcal{P} \quad (10)$$

From Lorentz invariance, we have [8]

$$-\mathcal{R}[\pm] = \lrcorner_{\mathcal{L}} \mathcal{K} + \lrcorner_{\mathcal{L}} \mathcal{P} \quad (11)$$

where, in the rest frame of the medium ($u_{\mu} = (1, \vec{0})$), the coefficients a_L, b_L are given by

$$a_L = \frac{1}{k^2} T_K - \frac{\omega}{k^2} T_u \quad (12)$$

$$b_L = \frac{\omega^2 - k^2}{k^2} T_u - \frac{\omega}{k^2} T_K \quad (13)$$

where $T_K = \frac{1}{4} Tr(\mathcal{K} \mathcal{R}[\pm])$ and $T_u = \frac{1}{4} Tr(\mathcal{U} \mathcal{R}[\pm])$.

In general, a_L, b_L are scalar functions of the invariants $x = K \cdot u = \omega$ and $y = K^2 = \omega^2 - k^2$; from (12) and (13) let us incidentally note that the coefficients a_L, b_L satisfy the following differential equations:

$$\frac{\partial b_L}{\partial y} + x \frac{\partial a_L}{\partial y} + \frac{\partial T_u}{\partial y} = 0 \quad (14)$$

$$\frac{\partial b_L}{\partial x} + x \frac{\partial a_L}{\partial x} + a_L + \frac{\partial T_u}{\partial x} = 0 \quad (15)$$

At first order in G_F , in (9),(10) the K -dependence vanishes, and then $a_L \simeq 0$. For b_L , the charged current contribution to ν_e self-energy is [8]

$$b_L^W \simeq -\sqrt{2} G_F (N_e - N_{\bar{e}}) \quad (16)$$

where $N_e(N_{\bar{e}})$ is the electron (positron) number density. For $\nu_e, \nu_{\mu}, \nu_{\tau}$ the neutral current contributions come from electron, proton and neutron background. Because the coupling to the Z^0 of the proton is opposite to that of the electron, for neutral media these two contributions cancel themselves [8] and one remains only with the neutron contribution:

$$b_L^Z \simeq \frac{G_F}{\sqrt{2}} (N_n - N_{\bar{n}}) \quad (17)$$

Then, in normal media (in which there are no antiparticles) we have

$$b_L \simeq -\sqrt{2}G_F(N_e - \frac{1}{2}N_n) \quad (18)$$

for ν_e , and

$$b_L \simeq \frac{G_F}{\sqrt{2}}N_n \quad (19)$$

for ν_μ, ν_τ .

Let us introduce now an external static magnetic field. We want to study coherent effects of electrons (and protons) in the background on neutrino propagation, so that the interaction region of the space in which we are interested must be microscopically large; however, this region is macroscopically small, and then we assume that the applied field is also uniform in space.

Here we don't consider the possible intrinsic neutrino magnetic moment which, in the standard model, is non-zero for massive neutrinos, but however extremely small [9]:

$$\mu_\nu \simeq \frac{3eG_F}{8\pi^2\sqrt{2}}m_\nu \simeq 3 \cdot 10^{-19} \left(\frac{m_\nu}{1\text{eV}} \right) \mu_B \quad (20)$$

where $\mu_B \simeq 3 \cdot 10^{-7} \text{eV}^{-1}$ is the Bohr magneton. The interaction of the neutrino with the magnetic field proceeds only through the interaction with the electrons and protons in the medium interacting with the field.

The extra-contributions to the neutrino self-energy come from the diagrams in fig. 3,4: for the electron background we have:

$$\begin{aligned} i\Sigma_W^B &= \lim_{\vec{q} \rightarrow 0} \int \frac{d^4P}{(2\pi)^4} \left(\frac{-ig}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5) \right) iS_F(P) (ie\gamma_\alpha) iS_F(P+Q) \cdot \\ &\cdot \left(\frac{-ig}{2\sqrt{2}} \gamma_\nu (1 - \gamma_5) \right) iD_W^{\mu\nu}(K-P) A^\alpha \end{aligned} \quad (21)$$

$$\begin{aligned} i\Sigma_Z^B &= \lim_{\vec{q} \rightarrow 0} \left(\frac{-ig}{4\cos\theta_W} \right) \gamma_\mu (1 - \gamma_5) iD_Z^{\mu\nu}(Q) \int \frac{d^4P}{(2\pi)^4} (-1) \cdot \\ &\cdot \text{Tr} \left[\left(\frac{ig}{4\cos\theta_W} \right) \gamma_\nu (1 - 4\sin^2\theta_W - \gamma_5) iS_F(P+Q) (ie\gamma_\alpha) iS_F(P) \right] A^\alpha \end{aligned} \quad (22)$$

(in the static limit, $Q_\mu = (0, \vec{q} \rightarrow 0)$). By taking into account only the temperature-dependent real part of the self-energy, with notation similar to (8), at lowest order in G_F we get (see appendix for calculation details)

$$\mathcal{R}[\widetilde{\pm \mathcal{B}}_W] = \lim_{\vec{q} \rightarrow 0} \frac{8eG_F}{\sqrt{2}} \int \frac{d^4P}{(2\pi)^4} \left[\frac{\Gamma_F(P)}{(P+Q)^2 - m_e^2} + \frac{\Gamma_F(P+Q)}{P^2 - m_e^2} \right] (T_{\alpha\mu}^S + iT_{\alpha\mu}^A) A^\alpha \quad (23)$$

$$\begin{aligned} \mathcal{R}[\widetilde{\pm \mathcal{Z}}_W] &= \lim_{\vec{q} \rightarrow 0} \left(-\frac{4eG_F}{\sqrt{2}} \right) \int \frac{d^4P}{(2\pi)^4} \left[\frac{\Gamma_F(P)}{(P+Q)^2 - m_e^2} + \frac{\Gamma_F(P+Q)}{P^2 - m_e^2} \right] \cdot \\ &\cdot ((1 - 4\sin^2\theta_W)T_{\alpha\mu}^S + iT_{\alpha\mu}^A) A^\alpha \gamma^\mu \end{aligned} \quad (24)$$

where the tensors $T_{\alpha\mu}^{S,A}$ are defined in the appendix. Again, from Lorentz invariance, we have that the extra-contribution to the neutrino self-energy can be written as

$$-\mathcal{R}]\widetilde{\pm\mathcal{B}} = -\mathcal{I}'_{\mathcal{L}} \mathcal{K} + \mathcal{I}'_{\mathcal{L}} \mathcal{I} + \mathcal{I}_{\mathcal{L}} \mathcal{B} \quad (25)$$

where $B_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu F^{\alpha\beta} = -i\epsilon_{\mu\nu\alpha\beta}u^\nu Q^\alpha A^\beta$ and, in the rest frame of the medium, $B_\mu = (0, \vec{B})$. To lowest order in G_F , the explicit K -dependence in (23), (24) again vanishes, and then $a'_L \simeq 0$. For the coefficients b'_L and c_L , in the appendix it is shown that, for symmetry reasons, also $b'_L \simeq 0$; the charged current contribution to c_L , instead, can be written as

$$c_L^W = -\frac{4eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \frac{d}{dE} (n_e - n_{\bar{e}}) \quad (26)$$

where $E = \sqrt{p^2 + m_e^2}$ is the electron energy. The neutral current contribution of the electron background can be obtained in a similar way (see appendix):

$$c_L^Z = \frac{2eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \frac{d}{dE} (n_e - n_{\bar{e}}) \quad (27)$$

Because of double sign change due to couplings with Z^0 and the magnetic field of electrons and protons, the neutral current contribution of the proton background is easily obtained by (27):

$$c_L^Z = \frac{2eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \frac{d}{dE} (n_p - n_{\bar{p}}) \quad (28)$$

where now $E = \sqrt{p^2 + m_p^2}$ is the proton energy, and n_p ($n_{\bar{p}}$) is the Fermi distribution function for protons (antiprotons).

Now, let us evaluate the integral

$$I_e = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \frac{d}{dE} n_e(E) \quad (29)$$

(the extension to protons is straightforward) in the two physical approximation outlined in the previous section.

Let us first consider the case of a degenerate electron gas ($T \ll T_F$); for this

$$\frac{d}{dE} n_e(E) \simeq -\delta(E - E_F) \quad (30)$$

where

$$E_F = m_e + \frac{p_F^2}{2m_e} = m_e + \frac{(3\pi^2 N_e)^{\frac{1}{3}}}{2m_e} \quad (31)$$

is the Fermi Energy, so that

$$I_e \simeq -\frac{m_e}{4\pi^2} \int_{m_e}^{+\infty} \frac{p(E)}{E} \delta(E - E_F) dE = -\frac{m_e}{4\pi^2} \frac{p_F}{E_F} \quad (32)$$

and then

$$I_e \simeq -\frac{p_F}{4\pi^2} \quad (33)$$

For a neutral medium, the Fermi momentum of protons is equal to that of electrons so that $I_p = I_e$.

Let us now consider a classical plasma, for which

$$\frac{d}{dE} n_e(E) \simeq -\beta n_e(E) \quad (34)$$

In the non-relativistic approximation for the electrons in the background,

$$\frac{1}{E} \simeq \frac{1}{m_e} \left(2 - \frac{E}{m_e}\right) \quad (35)$$

we have

$$I_e \simeq -\frac{\beta}{2m_e} \left(N_e - \frac{\overline{E}}{m_e}\right) \quad (36)$$

Where \overline{E} is the mean thermal electron energy, and is given by

$$\overline{E} = N_e m_e + \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{2m_e} n_e \simeq N_e \left(m_e + \frac{3}{4\beta}\right) \quad (37)$$

From (36) and (37) we obtain, for the electron background,

$$I_e \simeq \frac{3N_e}{8m_e^2} \quad (38)$$

Instead, the proton contribution is negligible:

$$I_p \simeq \frac{3N_p}{8m_p^2} \ll I_e \quad (39)$$

Finally, for a neutral medium, in the degenerate gas approximation we have got

$$c_L \simeq 0 \quad (40)$$

for ν_e and

$$c_L \simeq -\frac{eG_F}{\sqrt{2}} \frac{(3\pi^2 N_e)^{\frac{1}{3}}}{\pi^2} \quad (41)$$

for ν_μ, ν_τ , while in the approximation of a classical non-relativistic plasma

$$c_L \simeq -\frac{3eG_F}{4\sqrt{2}} \frac{N_e}{m_e^2} \quad (42)$$

for ν_e and

$$c_L \simeq +\frac{3eG_F}{4\sqrt{2}} \frac{N_e}{m_e^2} \quad (43)$$

for ν_μ, ν_τ . We now give an estimate of the order of magnitude of the previous results. For $N_e \sim 10^{23} \text{ cm}^{-3}$ we have $|c_L| \sim 10^{-22} \text{ eV}^{-1}$ for ν_μ, ν_τ in a degenerate gas, and $|c_L| \sim 10^{-27} \text{ eV}^{-1}$ for a classical plasma. Comparing the Wolfenstein term $|b_L| \sim 10^{-14} \text{ eV}$ with the energies

$$|c_L B| \sim 10^{-14} \left(\frac{B}{10^{10} \text{ Gauss}} \right) \text{ eV} \quad \text{from eq. (41)} \quad (44)$$

$$|c_L B| \sim 10^{-19} \left(\frac{B}{10^{10} \text{ Gauss}} \right) \text{ eV} \quad \text{from eqs. (42), (43)} \quad (45)$$

we deduce that these terms are relevant in most astrophysical (supernovæ, white dwarf, etc.) and cosmological environments, in which the magnetic field strength can be as large as $10^{10 \div 14} \text{ Gauss}$.

3 The dispersion relation.

The Dirac equation (5) has a non trivial solution only if the determinant $D(\omega, \vec{k}, \vec{B})$ of the 4x4 matrix $\mathcal{K} - m - \Sigma$ vanishes. The neutrino dispersion relation is then given by

$$D(\omega, \vec{k}, \vec{B}) = 0 \quad (46)$$

With Σ calculated in the previous section, at first order in G_F , the relation (46) reads

$$\omega^2 - k^2 = m^2 - b_L(\omega + k) + c_L(\omega + k) \frac{\vec{k} \cdot \vec{B}}{k} \quad (47)$$

for the usual left-handed neutrinos, and

$$\omega^2 - k^2 = m^2 - b_L(\omega - k) - c_L(\omega - k) \frac{\vec{k} \cdot \vec{B}}{k} \quad (48)$$

for the right-handed neutrinos. Observe that the presence of a privileged direction, that of the magnetic field \vec{B} , makes non-isotropic the dispersion relations (47),(48): besides a temporal dispersion in ω there is also a spatial dispersion in \vec{k} (also for isotropic media). Moreover, let us note that, on the contrary to that happens in absence of a magnetic field, making the transformation $k \rightarrow -k$ in (47),(48) these two relations don't transform each other; this happens only if one transforms \vec{B} in $-\vec{B}$. This point will be discussed in the next section.

For ultrarelativistic neutrinos, the left-handed ones propagate in matter with energy given by

$$\omega \simeq k + \frac{m^2}{2k} - b_L + c_L \frac{\vec{k} \cdot \vec{B}}{k} \quad (49)$$

while right-handed neutrinos, not having coherent interactions with matter, propagate as in vacuum:

$$\omega \simeq k + \frac{m^2}{2k} \quad (50)$$

The action of the thermal background on neutrino propagation can be described “macroscopically” by an effective potential V experienced by neutrinos. This potential can be defined [10] as the difference between the total energy and the kinetic energy of neutrinos, that is

$$V = -b_L + c_L \frac{\vec{k} \cdot \vec{B}}{k} \quad (51)$$

For $\vec{B} = 0$ we obtain the Wolfenstein result [3].

An interesting feature comes out for a particular configuration of the magnetic field, given by the relation

$$b_L k = c_L \vec{k} \cdot \vec{B} \quad (52)$$

As one can see from (49) or (51), if equation (52) holds, the coherent effects due properly to the magnetic field are opposite to those generated in absence of the field, and then the (left-handed) neutrinos propagate in matter as in vacuum. Let us examine the conditions under which (52) can be verified.

First of all, we observe that (52) is almost independent on neutrino energy; this dependence can occur only through b_L, c_L at order next to G_F . For propagation in a degenerate gas, from eqs. (18),(19) and (40),(41) we deduce that electron-neutrinos cannot experience the free propagation condition (52), while this condition can be verified by μ - and τ -neutrinos only if $\vec{k} \cdot \vec{B}$ is negative. Instead, all neutrino flavours can undergo the condition (52) in a non-relativistic classical plasma for $\vec{k} \cdot \vec{B}$ positive. Note that the free propagation condition doesn't take place for propagation normal to the magnetic field (in this case the magnetic field has no effect on the dispersion relation).

For ν_μ, ν_τ in a metal ($N_e \simeq 10^{23} \text{ cm}^{-3}$, $Z/A \simeq 1/2$) the field strength necessary to satisfy (52) scales as the power 2/3 of the density, and is approximatively $B \sim 10^8 \text{ Gauss}$. A very interesting thing happens, instead, for neutrino propagation in a classical non-

relativistic plasma: the field strength for (52) doesn't depend on medium characteristic ²

$$B \simeq \frac{|b_L|}{|c_L|} \simeq \frac{4}{3} \frac{m_e^2}{e} \quad (53)$$

Let us note that, apart a numerical factor, the field strength (53) corresponds to the Landau critical field $B_c = m^2/e \simeq 4.4 \cdot 10^{13} \text{ Gauss}$ [11] at which quantum effects become important in classical electrodynamics ³

4 Spatial dispersion.

In the previous section it has been pointed out that, also for isotropic media, because of the presence of a magnetic field, the dispersion relation (46) doesn't manifest spatial isotropy, but it depends on the relative direction of the neutrino momentum \vec{k} and the magnetic field \vec{B} (spatial dispersion). This means that the eigen-energies given by (47),(48) don't specify completely the eigen-modes of propagation of neutrinos in magnetized media: there are also eigen-directions to be picked out. To find these, we can follow two different ways.

A direct method consists in the application of the Ward identity at finite temperature [12] (this can be applied only for static fields). Let us rewrite the temperature-dependent real parts of the graphs in fig. 1,3 as

$$\begin{aligned} \mathcal{R}[\widetilde{\pm}_{\mathcal{W}}] &= \frac{g^2}{2} \int \frac{d^4 P}{(2\pi)^4} \frac{1}{P^2 - M_W^2} \gamma_\alpha (\not{P} + \not{K} + m_e) \gamma^\alpha \cdot \\ &\quad \cdot 2\pi n_F(P+K) \delta((P+K)^2 - m_e^2) \end{aligned} \quad (54)$$

$$\begin{aligned} \mathcal{R}[\widetilde{\pm}_{\mathcal{W}}^B] &= \frac{e g^2}{2} \int \frac{d^4 P}{(2\pi)^4} \frac{1}{P^2 - M_W^2} \gamma_\alpha (\not{P} + \not{K} + m_e) \gamma_\mu (\not{P} + \not{K} + m_e) \gamma^\alpha \cdot \\ &\quad \cdot 4\pi n_F(P+K) \frac{\delta((P+K)^2 - m_e^2)}{(P+K)^2 - m_e^2} A^\mu \end{aligned} \quad (55)$$

(for simplicity we consider a medium without antiparticles). Deriving (54) with respect to K_μ , and substituting in this expression the relation [12]

²A very weak dependence on medium is expressed in m_e , which would be the “effective” electron mass in matter.

³This limit can be obtained by equating $\hbar\omega_B = m_e c^2$ where $\omega_B = \frac{eB}{m_e c}$ is the gyrofrequency.

$$\begin{aligned}
- \frac{\partial}{\partial P_\mu} \left(2\pi n_F(P) \delta(P^2 - m_e^2) \right) &= (\not{P} + m_e) \gamma_\mu (\not{P} + m_e) \left[4\pi n_F(P) \frac{\delta(P^2 - m_e^2)}{P^2 - m_e^2} \right] + \\
&\quad - 2\pi \frac{\partial n_F(P)}{\partial P_\mu} (\not{P} + m_e) \delta(P^2 - m_e^2) , \tag{56}
\end{aligned}$$

for $A_\mu = (0, \vec{A})$, from (55) we obtain the following identity

$$\mathcal{R}[\widetilde{\pm \vec{B}}_{\mathcal{W}}] = -[\mathcal{A}_\mu \frac{\partial}{\partial \mathcal{K}_\mu} \mathcal{R}][\widetilde{\pm \mathcal{W}}(\mathcal{K})] \tag{57}$$

This relation is very similar to the usual Ward identity at $T = 0$ in Q.E.D. [13]; however, its applicability domain is much smaller than that at zero temperature (for a detailed discussion on the Ward identity at finite temperature see [12] and references therein). At first order in G_F , from (11) and (25) we deduce

$$\left(2e(K \cdot A) \left(\frac{\partial a_L}{\partial y} K_\mu + \frac{\partial b_L}{\partial y} u_\mu \right) + c_L B_\mu \right) \gamma^\mu \simeq 0 \tag{58}$$

From the independence of the Dirac matrices, we finally arrive to the following approximate equalities:

$$\frac{\partial b_L}{\partial y} + \omega \frac{\partial a_L}{\partial y} \simeq 0 \tag{59}$$

$$2e(K \cdot A) \frac{\partial a_L}{\partial y} \vec{k} \simeq c_L \vec{B} \tag{60}$$

The relation (59) is nothing that (14) at lowest order in G_F (at this order a straightforward calculation gives $\frac{\partial T_u}{\partial y} \simeq 0$). From (60), after simple manipulations, we can write ⁴

$$\frac{\vec{k}}{k} = \pm \frac{\vec{B}}{B} \tag{61}$$

Then, from (61), we deduce that the eigen-directions of neutrinos in the medium are the directions parallel and antiparallel to the applied magnetic field (the $+$ sign refers to ν_L while $-$ to ν_R).

The physical interpretation of this result comes again from the Dirac equation, that in the Weyl representation reads

$$(\omega + b_L + \vec{\sigma} \cdot (\vec{k} + c_L \vec{B})) \psi_L = m \psi_R \tag{62}$$

$$(\omega - \vec{\sigma} \cdot \vec{k}) \psi_R = m \psi_L \tag{63}$$

⁴At order G_F a direct computation gives $\frac{\partial a_L}{\partial y} \simeq \frac{G_F}{\sqrt{2}} \frac{\omega}{k^4} N_e$.

By defining the helicity eigen-states

$$\frac{\vec{\sigma} \cdot \vec{k}}{|\vec{k}|} \phi_\lambda = \lambda \phi_\lambda \quad (64)$$

with $\lambda = \pm 1$, because the interaction here considered through the diagrams in fig. 3,4 doesn't induce helicity flip ⁵, the spinors ψ_L, ψ_R must be proportional to ϕ_λ . From eq. (62) we deduce that this evenience can take place only if the operators $\vec{\sigma} \cdot \vec{k}$ and $\vec{\sigma} \cdot \vec{B}$ commute between them, that is (61) holds.

In other words, the neutrino eigen-directions, parallel to the magnetic field, are a direct consequence of helicity conservation of the interaction.

A peculiar feature of the spatial dispersion is the deflection of neutrino momentum at the exit of the medium. For evaluating this, let us consider the neutrino velocity \vec{v} , whose components are given by

$$v_i = \frac{\partial \omega}{\partial k_i} \quad (65)$$

From eq. (47) we then obtain

$$\vec{v} = \frac{1}{\omega} (\vec{k} + \vec{k}') \quad (66)$$

with

$$\vec{k}' = -b_L \frac{\vec{k}}{k} + c_L \vec{B} \quad (67)$$

For $\vec{B} = 0$ we recover the fact that there is no spatial dispersion (only the modulus of the velocity changes), while for \vec{B} and \vec{k} parallel between them there is no deflection, so that we reobtain the eigen-directions (61).

In general, for $\vec{k} \times \vec{B} \neq 0$ neutrinos are deflected from initial trajectory; if, for simplicity, we consider \vec{k} along the z-axis ($\vec{k} = (0, 0, k)$) and \vec{B} in the xz-plane ($\vec{B} = (B_x, 0, B_z)$) the angle θ between the final and the initial direction is given by

$$\tan \theta = \frac{v_x}{v_z} = \frac{c_L B_x}{k} \frac{1}{1 - \frac{b_L}{k} + \frac{c_L B_z}{k}} \simeq \frac{c_L B_x}{k} \ll 1 \quad (68)$$

In practice, for all physical situations this deflection is enormously small to detect.

5 Effects on neutrino matter oscillations.

In general if neutrinos are massive, the flavour (vacuum) eigen-states ν_α ($\alpha = e, \mu, \tau$) can be expressed as a linear superposition of the mass eigen-states ν_i ($i = 1, 2, 3$): $\nu_\alpha =$

⁵Terms in the self-energy that change helicity are of the form $d_L \frac{1-\gamma_5}{2} + d_R \frac{1+\gamma_5}{2}$, but in the Standard Model at order one-loop the coefficients d_L, d_R are vanishing (here we not consider intrinsic neutrino magnetic moment). In scenarios beyond the Standard Model, with more Higgs particles, these coefficients can be non zero also at order one-loop (see for example [14]).

$\sum_i U_{\alpha i} \nu_i$. Let us consider, for simplicity, the mixing of only two flavours (e and $\alpha = \mu$ or τ); in this case the mixing matrix U , if CP is conserved, can be cast in the form

$$U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (69)$$

where θ is the vacuum mixing angle. The hamiltonian of the sistem of two neutrinos that has to be considered for the propagation of a mass eigen-state in a magnetized medium has, in the ultrarelativistic approximation, the form

$$H = k + \frac{m^2}{2k} - b_L + c_L \frac{\vec{k} \cdot \vec{B}}{k} \quad (70)$$

where, in the mass eigen-states basis,

$$\frac{m^2}{2k} = \begin{pmatrix} \frac{m_1^2}{2k} & 0 \\ 0 & \frac{m_2^2}{2k} \end{pmatrix} \quad (71)$$

$$b_L = b_L^W \begin{pmatrix} \cos^2 \theta & -\sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} + b_L^Z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (72)$$

$$c_L = c_L^W \begin{pmatrix} \cos^2 \theta & -\sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} + c_L^Z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (73)$$

and b_L^W, b_L^Z are taken respectively by (16),(17) while c_L^W by (26) and c_L^Z by the sum of (27) and (28).

The eigen-energies of propagating neutrinos are given, then, by the eigen-values of H :

$$\begin{aligned} \omega_{1,2} = & k + \frac{1}{2} \frac{m_1^2 + m_2^2}{2k} - \frac{1}{2} \left(b_L^W - c_L^W \frac{\vec{k} \cdot \vec{B}}{k} \right) - \left(b_L^Z - c_L^Z \frac{\vec{k} \cdot \vec{B}}{k} \right) + \\ & \mp \frac{1}{2} \sqrt{\left(\frac{\Delta m^2}{2k} \cos 2\theta + b_L^W - c_L^W \frac{\vec{k} \cdot \vec{B}}{k} \right)^2 + \left(\frac{\Delta m^2}{2k} \sin 2\theta \right)^2} \end{aligned} \quad (74)$$

where $\Delta m^2 = m_2^2 - m_1^2$. For $\theta = 0$ we recover (49).

The matter eigen-states ν_{1m}, ν_{2m} again can be expressed as a linear superposition of the flavour eigen-states, with an effective mixing angle θ_m given by

$$\sin 2\theta_m = \frac{\frac{\Delta m^2}{2k} \sin 2\theta}{\sqrt{\left(\frac{\Delta m^2}{2k} \cos 2\theta + b_L^W - c_L^W \frac{\vec{k} \cdot \vec{B}}{k} \right)^2 + \left(\frac{\Delta m^2}{2k} \sin 2\theta \right)^2}} \quad (75)$$

Because of flavour superposition, if for example one creates a ν_e by weak interactions and then sends it in a magnetized medium, at the exit of the medium there is a non-vanishing

probability of finding a neutrino of another flavour. For a constant density medium, the transition probability at a certain distance x from the source is given by ⁶

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 \theta_m \sin^2 \frac{\pi x}{L_m} \quad (76)$$

where L_m is the effective oscillation length in matter,

$$L_m = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\pi}{\sqrt{\left(\frac{\Delta m^2}{2k} \cos 2\theta + b_L^W - c_L^W \frac{\vec{k} \cdot \vec{B}}{k}\right)^2 + \left(\frac{\Delta m^2}{2k} \sin 2\theta\right)^2}} \quad (77)$$

From (75) we observe that the MSW resonance condition is changed: in presence of a magnetic field the maximum effect of matter on neutrino oscillation (maximum effective mixing angle) occurs if the relation

$$\frac{\Delta m^2}{2k} \cos 2\theta = -b_L^W + c_L^W \frac{\vec{k} \cdot \vec{B}}{k} \quad (78)$$

holds. More explicitly, for a degenerate gas the resonance condition is

$$\frac{\Delta m^2}{2k} \cos 2\theta = \sqrt{2} G_F N_e + \frac{e G_F}{\sqrt{2}} \frac{(3\pi^2 N_e)^{\frac{1}{3}}}{\pi^2} \frac{\vec{k} \cdot \vec{B}}{k} \quad (79)$$

while for a classical non-relativistic plasma we have

$$\frac{\Delta m^2}{2k} \cos 2\theta = \sqrt{2} G_F N_e - \frac{3e G_F}{2\sqrt{2}} \frac{N_e}{m_e^2} \frac{\vec{k} \cdot \vec{B}}{k} \quad (80)$$

The effect of the magnetic field, as it has already been pointed out, vanishes for propagation normal to the field and, in the other cases, is important for strong fields.

A novel feature appears when the magnetic field configuration is such that

$$b_L^W k = c_L^W \vec{k} \cdot \vec{B} \quad (81)$$

In this case, neutrinos don't propagate freely in matter ⁷ but no resonance occurs. In fact, if (81) holds, the hamiltonian (70) is diagonal, so that no effective mixing angle θ_m arises. The conditions under which (81) can be verified are similar to those obtained in section 3.

⁶The calculation of the transition probability proceeds as in the MSW theory without a magnetic field, but with an effective mixing angle given by (75). See for example [15].

⁷From (70), for free propagation in matter, not only (81) but also the relation

$$\frac{b_L^W}{b_L^Z} = \frac{c_L^W}{c_L^Z} \quad (82)$$

between the coefficients must be realized.

6 Discussions and conclusions.

In the present work we have studied the modification of the neutrino “effective properties” in an isotropic medium when a magnetic field is present. Because of the privileged direction introduced by the magnetic field \vec{B} , spatial dispersion arises so that for individuating the eigen-modes of neutrino propagation one has to recognize also the eigen-directions of neutrinos. From the helicity conservation in the interaction between neutrinos and the magnetized matter, we have that the eigen-directions are those parallel to the magnetic field. A direct consequence of this spatial dispersion is the deflection of the neutrino momentum at the exit of the medium; however, a simple calculation (see (68)) shows that the deflection is practically no detectable in all physical situations.

An important result is the modification of the effective potential (51) experienced by neutrinos in matter under the influence of the magnetic field. For neutrino matter oscillations, this implies that the MSW resonance condition changes according to (78) for sufficiently high magnetic fields. This can be easily visualized with the level crossing diagrams showed in figs. 5-7. The resonance point change with the field strength and with the field direction, reaching maximum modification for neutrino propagation along the magnetic field, while no modification occurs for normal propagation. In particular, if the field configuration is such that the charged current contribution to neutrino effective potential cancel themselves, no difference arises in the propagation of the two neutrino eigen-states, so that no level crossing occurs and then no resonance can be present. Moreover, for this field configuration, if we consider a (neutral) degenerate gas with

$$N_e - N_{\bar{e}} = \frac{1}{2} (N_n - N_{\bar{n}}) \quad (83)$$

or a classical non-relativistic plasma with

$$N_e - N_{\bar{e}} = (N_n - N_{\bar{n}}) \quad (84)$$

the medium is completely transparent to neutrinos (see (82)). Let us note that the no resonance condition (81) and the additional relation (82) for free propagation are independent on neutrino energy. Furthermore, for those physical situations in which the classical plasma approximation is valid, the strength of the magnetic field necessary to satisfy (81) doesn’t depend on medium characteristic but, apart a numerical factor, coincides with the Landau critical field $B_c = \frac{m_e^2}{e} \simeq 4.4 \cdot 10^{13} Gauss$.

These considerations should be kept in mind when one studies neutrino propagation in astrophysical and cosmological environments. In fact, for example, even if one ignores specific details, a magnetic field is present in supernovæ with strengths up to $10^{14} Gauss$ [16]: this implies that resonant MSW oscillations of neutrinos through their interactions with background electrons, protons and neutrons (but not with background neutrinos) can no longer occur. Moreover, the possibility of completely free propagation in these backgrounds can alter significantly the dynamics of the supernovæ.

Instead, MSW oscillations of solar neutrinos [5] are practically no interested by our results because of too small magnetic fields present in the Sun [17].

Another physical context in which one can use our results is the evolution of the Universe. In fact, a magnetic field close to the critical field B_c is present [18] just before the epoch of the nucleosynthesis of the light elements, and the number of each type of neutrinos is important for the development of the nucleosynthesis itself [19].

Appendix

A Calculation details.

Let us evaluate the neutrino self-energy in a magnetized medium by starting with charged-current diagram in fig. 3. We follow the lines traced by D'Olivio, Nives and Pal in Ref. [3].

At lowest order in G_F , the temperature-dependent real part of the self-energy is given by:

$$\begin{aligned} \mathcal{R}[\pm \mathcal{B}_{\mathcal{W}}] &= - \lim_{|\vec{q}| \rightarrow 0} \frac{4eG_F}{\sqrt{2}} \int \frac{d^4 P}{(2\pi)^4} \gamma_\mu \frac{1 - \gamma_5}{2} (\not{P} + \not{Q} + m_e) \gamma_\alpha (\not{P} + m_e) \gamma^\mu \frac{1 - \gamma_5}{2} \cdot \\ &\cdot \left[\frac{\Gamma_F(P)}{(P + Q)^2 - m_e^2} + \frac{\Gamma_F(P + Q)}{P^2 - m_e^2} \right] A^\alpha \end{aligned} \quad (85)$$

By writing

$$\gamma_\mu \frac{1 - \gamma_5}{2} (\not{P} + \not{Q} + m_e) \gamma_\alpha (\not{P} + m_e) \gamma^\mu \frac{1 - \gamma_5}{2} = -T_{\alpha\mu} \gamma^\mu \frac{1 - \gamma_5}{2} \quad (86)$$

with

$$T_{\alpha\mu} \equiv \text{Tr} \left[(\not{P} + \not{Q} + m_e) \gamma_\alpha (\not{P} + m_e) \gamma^\mu \frac{1 - \gamma_5}{2} \right] = 2 (T_{\alpha\mu}^S + i T_{\alpha\mu}^A) \quad (87)$$

$$T_{\alpha\mu}^S = 2P_\alpha P_\mu + Q_\alpha P_\mu + P_\alpha Q_\mu - (P^2 - m_e^2) g_{\alpha\mu} - P \cdot Q g_{\alpha\mu} \quad (88)$$

$$T_{\alpha\mu}^A = \epsilon_{\alpha\mu\delta\beta} P^\delta Q^\beta \quad (89)$$

one obtains:

$$\begin{aligned} \mathcal{R}[\widetilde{\pm \mathcal{B}_{\mathcal{W}}}] &= \lim_{|\vec{q}| \rightarrow 0} \frac{8eG_F}{\sqrt{2}} \int \frac{d^4 P}{(2\pi)^4} 2\pi \delta(P^2 - m_e^2) [\theta(P^0) n_e(P) + \theta(-P^0) n_{\bar{e}}(P)] \cdot \\ &\cdot \left[\left(\frac{T_{\alpha\mu}^S(Q)}{P^2 - m_e^2 + Q^2 + 2P \cdot Q} + (Q \rightarrow -Q) \right) + \right. \\ &\left. + i T_{\alpha\mu}^A(Q) \left(\frac{1}{P^2 - m_e^2 + Q^2 + 2P \cdot Q} + (Q \rightarrow -Q) \right) \right] A^\alpha \gamma^\mu \end{aligned} \quad (90)$$

Eliminating the δ -function with the energy integration, we have

$$\begin{aligned} \mathcal{R}[\widetilde{\pm \mathcal{B}_{\mathcal{W}}}] &= \lim_{|\vec{q}| \rightarrow 0} \frac{8eG_F}{\sqrt{2}} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E} \cdot \\ &\cdot \left[(n_e + n_{\bar{e}}) \left(\frac{2P_\alpha P_\mu + Q_\alpha P_\mu + P_\alpha Q_\mu - P \cdot Q g_{\alpha\mu}}{Q^2 + 2P \cdot Q} + (Q \rightarrow -Q) \right) + \right. \\ &\left. + (n_e - n_{\bar{e}}) i \epsilon_{\alpha\mu\delta\beta} P^\delta Q^\beta \left(\frac{1}{Q^2 + 2P \cdot Q} + (Q \rightarrow -Q) \right) \right] A^\alpha \gamma^\mu \end{aligned} \quad (91)$$

The dispersion relation must be electromagnetic gauge-invariant; to ensure this, let us introduce the 4-vector $B_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu F^{\alpha\beta}$ that in the rest frame of the medium has component $B_\mu = (0, \vec{B})$. From Lorentz invariance, because (91) depends only on the 4-vectors u_μ , B_μ , one can write

$$-\mathcal{R}[\pm\vec{B}] = \mathcal{L}' \mathcal{A} + \mathcal{L} \mathcal{B} \quad (92)$$

where, from $u \cdot B = 0$, the coefficients are given by

$$b'_L = -\frac{1}{4} \text{Tr}(\mathcal{A} \mathcal{R}[\pm\vec{B}]) \quad (93)$$

$$c_L = -\frac{1}{4B^2} \text{Tr}(\mathcal{B} \mathcal{R}[\pm\vec{B}]) \quad (94)$$

From gauge-invariance we have that $Q \cdot A = 0$ and, for a static field, $Q \cdot u = 0$. Moreover, by choosing $A_\mu = (0, \vec{A})$, also $u \cdot A = 0$ holds. From these, the coefficient b'_L can be written as

$$\begin{aligned} b'_L = & \lim_{|\vec{q}| \rightarrow 0} \frac{8eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} \left[(n_e + n_{\bar{e}}) \vec{p} \cdot \vec{A} + \right. \\ & \left. + (n_e - n_{\bar{e}}) \frac{\vec{p} \cdot \vec{B}}{2E} \left(\frac{1}{2\vec{p} \cdot \vec{q} - \vec{q}^2} - \frac{1}{2\vec{p} \cdot \vec{q} + \vec{q}^2} \right) \right] \end{aligned} \quad (95)$$

Let us first perform the integration on the azimuthal θ -angle; because the integrand function is odd under the exchange $\cos \theta \rightarrow -\cos \theta$, the integration yields

$$b'_L = 0 \quad (96)$$

For the coefficient c_L one instead gets:

$$\begin{aligned} c_L = & \lim_{|\vec{q}| \rightarrow 0} \frac{8eG_F}{\sqrt{2}} \left[\int \frac{d^3p}{(2\pi)^3} (n_e + n_{\bar{e}}) \frac{(\vec{p} \cdot \vec{A})(\vec{p} \cdot \vec{B})}{B^2} \frac{1}{E} \left(\frac{1}{2\vec{p} \cdot \vec{q} - \vec{q}^2} - \frac{1}{2\vec{p} \cdot \vec{q} + \vec{q}^2} \right) + \right. \\ & \left. - \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} (n_e - n_{\bar{e}}) \left(\frac{1}{2\vec{p} \cdot \vec{q} - \vec{q}^2} - \frac{1}{2\vec{p} \cdot \vec{q} + \vec{q}^2} \right) \right] \end{aligned} \quad (97)$$

In the first integral, the integrand function is again odd in $\cos \theta$, so that one remains with

$$c_L = - \lim_{|\vec{q}| \rightarrow 0} \frac{4eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} (n_e - n_{\bar{e}}) \left(\frac{1}{2\vec{p} \cdot \vec{q} - \vec{q}^2} - \frac{1}{2\vec{p} \cdot \vec{q} + \vec{q}^2} \right) \quad (98)$$

Because the function in the integral of eq. (98) is uniformly sommable, we can perform first the limit $\vec{q} \rightarrow 0$ (fixed the direction of \vec{q}); for this purpose let us make the substitutions $\vec{p} \rightarrow \vec{p} + \frac{1}{2}\vec{q}$ in the first term of (98) and $\vec{p} \rightarrow \vec{p} - \frac{1}{2}\vec{q}$ in the second one:

$$c_L = - \frac{4eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} \lim_{|\vec{q}| \rightarrow 0} \frac{1}{2\vec{p} \cdot \vec{q}} ([n_e(E_+) - n_e(E_-)] - [n_{\bar{e}}(E_+) - n_{\bar{e}}(E_-)]) \quad (99)$$

Neglecting terms of the second order in $|\vec{q}|$,

$$E_{\pm} \equiv \sqrt{\left(\vec{p} \pm \frac{1}{2}\vec{q}\right)^2 + m_e^2} \simeq E \pm \frac{1}{2} \frac{\vec{p} \cdot \vec{q}}{E} \quad (100)$$

one obtains

$$c_L = -\frac{4eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} \left[\lim_{h \rightarrow 0} \frac{n_e(E + \frac{1}{2}h) - n_e(E - \frac{1}{2}h)}{h} + \right. \quad (101)$$

$$\left. - \lim_{h \rightarrow 0} \frac{n_{\bar{e}}(E + \frac{1}{2}h) - n_{\bar{e}}(E - \frac{1}{2}h)}{h} \right] \quad (102)$$

Performing the limits, we finally arrive at

$$c_L = -\frac{4eG_F}{\sqrt{2}} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \frac{d}{dE} (n_e - n_{\bar{e}}) \quad (103)$$

that is eq.(26).

For the neutral-current contribution (for example from electron background), from the identity (86) we have

$$\mathcal{R}[\widetilde{\pm \frac{\mathcal{B}}{Z}}] = - \lim_{|\vec{q}| \rightarrow 0} \frac{\epsilon[\mathcal{G}_{\mathcal{F}}]}{\sqrt{\epsilon}} \int \frac{[\Delta \mathcal{P}]}{(\epsilon \pi)^{\Delta}} \left[\frac{-\mathcal{F}(\mathcal{P})}{(\mathcal{P} + \mathcal{Q})^{\epsilon} - \mathbb{J}_{\uparrow}^{\epsilon}} + \frac{-\mathcal{F}(\mathcal{P} + \mathcal{Q})}{\mathcal{P}^{\epsilon} - \mathbb{J}_{\uparrow}^{\epsilon}} \right] \mathcal{T}'_{\alpha\mu} \mathcal{A}^{\alpha} \gamma^{\mu} \quad (104)$$

where

$$T'_{\alpha\mu} = 2 \left[(1 - 4 \sin^2 \theta_W) T_{\alpha\mu}^S + i T_{\alpha\mu}^A \right] \quad (105)$$

and $T_{\alpha\mu}^{S,A}$ are given by (88),(89). From these, it is simple to observe that also the neutral-current contribution to b'_L vanishes while, because the symmetric part of $T'_{\alpha\mu}$ always vanishes in the integration over $\cos \theta$, the contribution to c_L can be obtained by the charged-current one by substituting G_F with $-G_F/2$.

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Figure 1: *Charged-current Feynman diagram for ν_e self-energy in a non-magnetized medium.*

Figure 2: *Neutral-current Feynman diagram for ν_e, ν_μ, ν_τ self-energy in a non-magnetized medium.*

Figure 3: *Charged-current Feynman diagram for ν_e self-energy in a magnetized medium.*

Figure 4: *Neutral-current Feynman diagram for ν_e, ν_μ, ν_τ self-energy in a magnetized medium.*

Figure 5: *Level crossing diagram for 1 Mev momentum neutrino propagation normal to the magnetic field (this diagram coincides with that for $\vec{B} = 0$): the continuous curves refer to $\omega_{1,2}$ in eq.(74), while the dashed lines to $\omega_{1,2}$ with zero mixing angle. We use the typical values: $m_1 \simeq 10^{-8} \text{ eV}$, $m_2 \simeq 10^{-3} \text{ eV}$, $\sin^2 2\theta \simeq 7 \cdot 10^{-3}$.*

Figure 6: *Level crossing diagram for 1 Mev momentum neutrino propagation in a degenerate Fermi gas parallel to a magnetic field with strength $B \simeq 10^8 \text{ Gauss}$. We use the same values of fig.5 for neutrino masses and vacuum mixing angle.*

Figure 7: *Level crossing diagram for 1 Mev momentum neutrino propagation in a classical plasma parallel to a magnetic field with strength $B \simeq 10^{14} \text{ Gauss}$. We use the same values of fig.5 for neutrino masses and vacuum mixing angle.*

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